

The Effect of Air Resistance

Harvey Gould

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I. INTRODUCTION

I simulated the motion of a falling body near the earth's surface in the presence of gravity and air resistance. I used the Euler, Euler-Cromer, and Euler-Richardson algorithms and considered both a linear and quadratic dependence of the drag force on the velocity. Of particular interest is the rise and fall of a pebble and the relative times of ascent and descent.

II. METHOD

As shown in the text, Newton's laws of motion can be written as two coupled first-order differential equations. The Euler algorithm for the numerical solution of the latter equations can be written in the form

$$v_{n+1} = v_n + a_n \Delta t \quad (1a)$$

$$y_{n+1} = y_n + v_n \Delta t. \quad (1b)$$

The Euler-Cromer algorithm modifies Eq. (1) slightly and can be expressed as

$$v_{n+1} = v_n + a_n \Delta t \quad (2a)$$

$$y_{n+1} = y_n + v_{n+1} \Delta t. \quad (2b)$$

A more sophisticated version of the Euler algorithm evaluates the acceleration and the velocity at the time $\Delta t/2 = t + \Delta t/2$. The Euler-Richardson algorithm can be written as follows:

$$a_m = F(y_m, v_m, \Delta t/2)/m \quad (3a)$$

$$v_m = v_n + a_m \Delta t/2 \quad (3b)$$

$$y_m = y_n + v_m \Delta t/2, \quad (3c)$$

and

$$v_{n+1} = v_n + a_m \Delta t \quad (4a)$$

$$y_{n+1} = y_n + v_m \Delta t. \quad (4b)$$

The total force on a falling body in the presence of air resistance can be written in the convenient form

$$F = -mg(1 - v^2/v_t^2) \quad (\text{quadratic drag force}) \quad (5a)$$

or

$$F = -mg(1 - v/v_t) \quad (\text{linear drag force}) \quad (5b)$$

The signs in Eq. (5) are for a falling body with the vertical coordinate increasing upward.

III. VERIFICATION

I made various modifications of so that I could determine whether the Euler, Euler-Cromer, or Euler-Richardson gave the most accurate results for the position of a freely falling body. My numerical results using the three algorithms are summarized in Table I. Note that the three algorithms give the exact result for the velocity because the acceleration is constant for free fall. We see that the Euler-Richardson algorithm gives the most accurate result for a given value of Δt . For this reason I will adopt the Euler-Richardson algorithm with $\Delta t = 0.001$ for the rest of this report. I also note that the Euler and Euler-Cromer algorithms overestimate and underestimate the exact answer, respectively, and that their average value gives the exact answer for y .

| Δt | y (Euler) | y (Euler-Cromer) | y (Euler-Richardson) |
|------------|-------------|--------------------|------------------------|
| 0.1 | 5.5900 | 4.6100 | 4.8550 |
| 0.01 | 5.1490 | 5.0510 | 5.0755 |
| 0.001 | 5.1049 | 5.0951 | 5.0976 |

TABLE I: Comparison of the results of the Euler, Euler-Cromer, and Euler-Richardson algorithms with the analytical result, $y(t) = y_0 - v_0 t - gt^2/2 = 5.1$ at $t = 1$ with $y_0 = 10$ and $v_0 = 0$. Note that the Euler-Richardson algorithm gives the most accurate result for a given value of Δt .

IV. DATA

My first goal is to try to determine the form of the velocity dependence of the drag resistance on a styrofoam ball. To do so I will first use the data in Table II (the same as Table 3.1 in the text) to try to find the velocity dependence of the acceleration directly. The velocity and acceleration were found from the approximations

$$v(t) = [y(t + \Delta t) - y(t)]/\Delta t \quad (6)$$

$$a(t) = [y(t + \Delta t) - 2y(t) + y(t - \Delta t)]/(\Delta t)^2. \quad (7)$$

From Table II we see that it is impossible to determine the velocity-dependence of the acceleration. The reason is that an error is introduced in trying to use the approximate formulas (6) and (7) to estimate v and a . For this reason I now assume a functional form of the velocity dependence and try to find the best fit to the measured values of the position given in Table II.

| time | position | velocity | acceleration |
|------|----------|----------|--------------|
| 0 | 0.075 | – | – |
| 0.1 | 0.26 | 2.65 | 8 |
| 0.2 | 0.525 | 3.45 | 8 |
| 0.3 | 0.870 | 4.00 | 5.5 |
| 0.4 | 1.27 | 4.60 | 6 |
| 0.5 | 1.73 | 5.00 | 4 |
| 0.6 | 2.23 | 5.40 | 4 |
| 0.7 | 2.77 | 5.80 | 4 |
| 0.8 | 3.35 | – | – |

TABLE II: The derived values of the velocity and the acceleration from the empirical values of the position y . The values of y are taken from Table 3.1 of the text. Note that the acceleration does not have an obvious dependence on the velocity.

I now use drag to determine the best fit to the empirical data. My results are summarized in Fig. 1.

I wrote the class `Pebble` so that I could simulate the time for a pebble to ascend and

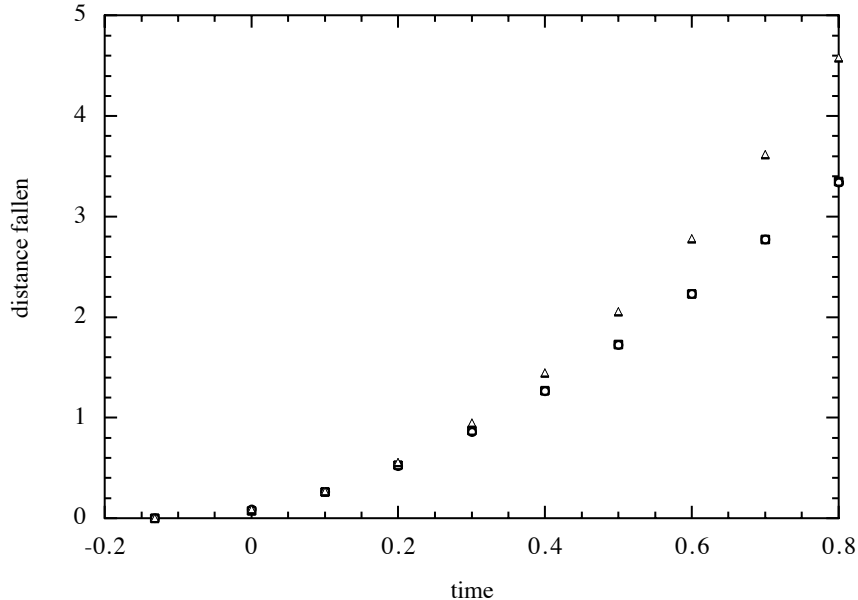


FIG. 1: Comparison of the measured distance (open squares) fallen by a styrofoam ball with the calculated distance assuming a quadratic drag (open circles) and a linear drag (open triangles). The terminal velocity was assumed to be 6.5 m/s in both cases. Note that the quadratic form fits much better.

descend. The main programming change I made was to use the `Math.abs` method to account for the fact that the drag force changes sign.

I assumed a quadratic drag force and that the ball is thrown vertically upward with an initial velocity $v_0 = 50$. The drag force can be written in terms of the terminal velocity even if the ball is moving upwards. I assumed that $v_t = 30$, a value much bigger than the value for the styrofoam ball. I found that the time up was approximately 3.15 and the time down was 3.93. I also found that the speed of the ball when it returned to the earth was 25.73, a value much less than the initial speed. All units are SI.

V. ANALYSIS

I did not try to optimize the value of the terminal velocity and used the Euler-Richardson algorithm with $\Delta t = 0.001$ for all runs. Because it was clear from my results (see Fig. 1) that the quadratic form of the drag resistance fits the empirical data better than the linear form, no more analysis was really necessary.

VI. INTERPRETATION

I initially thought that the time up would be shorter than the time down because the pebble starts out with a big velocity. I was somewhat surprised to find that my intuition was correct. The ball reached a height of 61.03 m, much less than the height of $v_0^2/2g = 127.6$ m it would have reached in the absence of drag resistance.

VII. CRITIQUE

It was fun to model the behavior of a system that I almost understand. I was surprised that I understood that the time up would be less than the time down. I would have liked to simulate two- dimensional trajectories so I could have used my newfound knowledge of Java's graphics capabilities, but I didn't have time.